

1. Motion

Physics.

1. Mass. doesn't change if you move it to a new place even another planet.

2. Displacement: it is a vector has direction. m.
$$\vec{\Delta S} = \vec{S}_2 - \vec{S}_1$$

3. Velocity: $v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{\Delta t}$ (Uniform Motion) m/s

average velocity: $v_{avg} = \frac{\vec{\Delta S}}{\Delta t}$ (velocity changes during ΔS)

special: $v_{avg} = \frac{v_1 + v_2}{2}$ (Uniform Acceleration)

4. Acceleration: $\vec{a} = \frac{\vec{\Delta v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ m/s²

5. Uniform Acceleration: $v_{avg} = \frac{v_1 + v_2}{2}$

$$\Delta x = v_{avg} \cdot \Delta t = \frac{(v_1 + v_2) \cdot t}{2}$$

$$\Delta x = v_1 \cdot \Delta t + \frac{1}{2} a \cdot \Delta t^2$$

$$v_2 = v_1 + a \cdot \Delta t$$

$$v_2^2 - v_1^2 = 2a \cdot \Delta x$$

2. Laws of Motion

1. First law of motion:

If the force on an object are balanced, then the object moves with constant velocity. (in a straight line).

If the net force on an object is not zero, the object can not be undergoing uniform motion. It is either speeding up / slowing down / *changing direction.

2. Second Law of motion:

$$\vec{F}_{\text{net}} = \vec{a} \cdot m$$

$$N. = \text{kg} \cdot \text{m} / \text{s}^2.$$

3. Third Law of motion:

To every action there is an equal & opposite reaction.

If object 1 exerts a force \vec{F}_{12} on object 2.

Then object 2 exerts a force \vec{F}_{21} on object 1.

$$\vec{F}_{12} = -\vec{F}_{21}$$

3. Gravitation .

1. The law of gravitation .

$$F_{\text{grav}} = \frac{G m_1 m_2}{d^2}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

on the surface of earth: $F_{\text{grav}} = m \cdot g$

2. Horizontal & Vertical Motion :

Horizontal & Vertical Motion are independent .

$$(F_{\text{net}})_y = m \cdot a_y$$

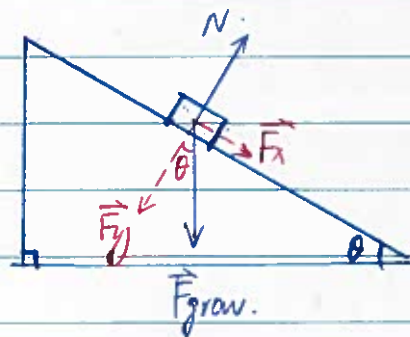
$$\Delta y = \frac{1}{2} (v_{iy} + v_{fy}) \cdot \Delta t$$

$$(F_{\text{net}})_y = F_{iy} + F_{iy} + \dots$$

$$a_y = F_y/m$$

$$(F_{\text{net}})_x = F_{ix} + F_{ix} + \dots$$

$$a_x = F_x/m$$



4 Planes & Circles

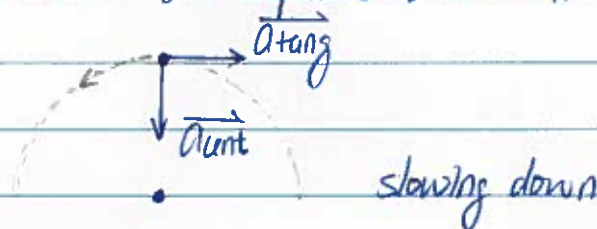
1. Circular Motion.

If an object is moving at constant speed in a circle, then the net force on the subject points toward the center of the circle and a vector also points toward the center.

$$\star a_{\text{cent}} = \frac{v^2}{r}$$

$$a_{\text{tang}} = \frac{\Delta v}{\Delta t}$$

$$v = \frac{2\pi R}{T}$$



5. Friction & Air Resistance

1. Static friction:

No slipping surface. $a_x = 0$, $a_y = 0$.

$$F_{s\max} = \mu_s \cdot N$$

2. Kinetic friction:

slipping between surfaces.

$$F_k = \mu_k \cdot N$$

* 与物体滑动的 v 无关

* 与接触面积 A 无关

3. Air Resistance

$$F_{\text{drag}} = C \cdot \rho \cdot A \cdot v^2$$

$C = 0.2$ ρ density of the fluid/air.

A cross-section area of the moving object.

4. Determine whether air resistance can be neglected:

1. 先假设可忽略, 计算出 v

2. 将 v 值代入 $F_{\text{drag}} = C \cdot \rho \cdot A \cdot v^2$ 得到 F_{drag} 值

3. 比较 F_{drag} 与 F . 若 $F_{\text{drag}}/F \ll 1$ 则可忽略.

6. Torques

1. Moment of inertia of a ring shape object:

$$I = M \cdot R^2$$

半径大的物体, 不易被转动

2. Spinning wheel. frequency $f = 1/T$

3. Torques: $T = r \cdot F \cdot \sin\phi$

P_0 pivot, the point of axis

P_1 受力点



counterclockwise: +, clockwise: -

4. Equilibrium. ★

$$(F_{net})_x = 0$$

$$(F_{net})_y = 0$$

$$T_{net} = 0$$

no matter which point you choose as the pivot

♥ 未知 - 种力的大小时, 选择该力可产生 T 的点为 pivot. eg. 未知 T:

I. A as the pivot.

可对 A 点产生 torques 的力有: \vec{T} , $m_1 g$, $m_2 g$

\therefore the system is equilibrium $\therefore T_{net} = 0$

$$\therefore T_{net} = l \cdot \sin\theta \cdot T - \frac{1}{2} l \cdot m_1 g - l \cdot m_2 g = 0$$

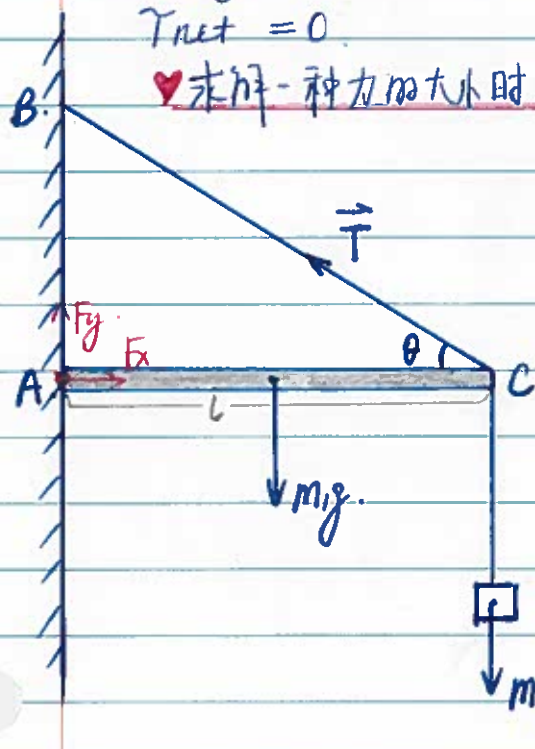
$$\therefore T = \frac{m_1 g + 2m_2 g}{2 \sin\theta}$$

eg. 未知 vertical force exerted by the wall on the pole (F_y)

II. C as the pivot.

可对 C 产生 T 的力有 \vec{F}_y , $m_1 g$

$$T_{net} = l \cdot \vec{F}_y - \frac{1}{2} l \cdot m_1 g = 0$$



7 Solid Properties :

1. For object made with same material :

A force will stretch a long wire more than a short wire.

$$\text{strain} = \Delta L / L$$

A force will stretch a thin wire more than a thick wire.

$$\text{stress} = F / A$$

eg. A force F on the two ends of rod A which length is L will cause ΔL ; when apply F on the two ends of rod B, which length is $3L$, will cause elongation $\sim 3\Delta L$.

For the two ends of rod A with radius r cause ΔL .

For the two ends of rod B (the same length as A) with $2r$ will cause $1/4 \Delta L$ (\because in this case $A = \pi r^2$).

2. Scale models

When the linear dimensions of a structure are all \uparrow by a factor x , the cross section area \uparrow by x^2 , the volume & mass \uparrow by x^3 .
the stress $= F/A \uparrow$ by x .

8. Momentum

1. momentum of a single object

$$\vec{p} = m \cdot \vec{v}$$

2. Conservation of momentum:

If a system of objects is isolated (external forces are balanced) then the total momentum of the system stays constant.

$$\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$$

of collision

3. External forces & Impulse

When the external forces are not balanced:

$$\Delta \vec{p} = \vec{p}_2 - \vec{p}_1$$

$$\vec{F}_{\text{net}} = m \cdot \vec{a} = m \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{m\vec{v}_2 - m\vec{v}_1}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\therefore \Delta \vec{p} = \vec{F}_{\text{net}} \times \Delta t$$

9. Energy

1. Work: $W = F \cdot \Delta x \cdot \cos \phi$.

ϕ is the angle between the direction of \vec{F} & the direction of the displacement $\vec{\Delta x}$.

$$W_{\text{tot}} = F_{\text{net}} \cdot \Delta x \cdot \cos \phi.$$

* F 与位移方向相同时 $\cos \phi = 1$ 相反时 $\cos \phi = -1$ 垂直时 $\cos 90^\circ = 0$.

2. Energy of Motion:

$$W = F \cdot \Delta x = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta E_k$$

$$F = ma = \frac{m\Delta v}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t}$$

$$\Delta x = \frac{v_2 + v_1}{2} \cdot \Delta t$$

$$E_k = \frac{1}{2}mv^2.$$

3. Potential Energy: $E_p = mgh$.

4. Conservation of Energy:

eg. $E_{k1} + E_{p1} = E_{k2} + E_{p2}$.

5. Efficiency of Energy Conversion.

$$\text{Efficiency} = \frac{\text{Energy in desired form}}{\text{Energy in original form}} \times 100\%$$

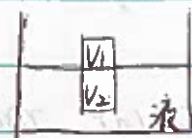
6. Power: $P = \frac{\Delta E}{\Delta t}$

7. Pulleys:
1. 在一根绳上 the tension is the same along the rope.
 2. rope pulled @ v by hand = on some load.

10. Fluids

1 Density $\rho = \frac{m}{V}$ mass/volume

$$\text{Specific gravity} = \frac{\rho}{\rho_{H_2O}}$$



$$\frac{V_1}{V_2} = \frac{\rho_{H_2O} - \rho_{obj}}{\rho_{obj}}$$

$$\rho_{H_2O} = 1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$$

2 Pressure $P = F/A$ N/m^2

$$1 \text{ N/m}^2 = 1 \text{ Pa}$$

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 14.7 \text{ psi} = 760 \text{ mmHg}$$

3 Buoyant force: $F_B = \rho \cdot V \cdot g$

* ρ : density of the fluid

V : volume of the displaced fluid

4 Law of Hydrostatic Equilibrium: In a body of fluid:

① the pressure at 2 points separated vertically by height h

$$P_2 = P_1 + \rho g h$$

② the pressure at 2 points separated horizontally are the same

③ the pressure at a given point is the same in all directions

5 Gauge Pressure: $P_{\text{gauge}} = P - P_{\text{atm}}$

6 Surface Tension: $F_{\text{max}} = \gamma \cdot L^*$

* L 为浸入水中两边缘长度

eg. ① A needle floats on the surface of the water.
length = 3cm. width very small. F_{\max} by the surface tension of water?

$$L = 2(l+w) \approx 2l = 6\text{cm}.$$

针的周长.

② A wire circle. radius: r . F_{\max} ?

$$L = 2 \cdot C = 2 \times 2\pi r = 4\pi r$$



圆外周接触水.

③ A solid circle. radius: r . F_{\max} ?

$$L = C = 2\pi r$$



仅外周接触水.

7 Continuity: The flow rate f of the water in the same flow eg. river is the same at each point along the flow.

$$f_1 = f_2$$

$$f = A \cdot v$$

* A : area of the cross-section

v : flow speed / velocity.

8. Viscosity & Turbulence.

$$\text{Friction} = \eta \cdot \frac{A v}{d}$$

$$Re = \frac{L \cdot \rho v}{\eta}$$

* η : viscosity in kg/m.s .

d : depth of the fluid.

* L : size of the obstacle in the flow. m .

ρ : density of the fluid, kg/m^3 .

v : flow velocity m/s .

* Re 越大 流体越稀 越易 turbulent

9. Bernoulli's Principle:

For incompressible, laminar, inviscid flow. if point 1 & 2 are on the same streamline:

$$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\underline{P \cdot \Delta V} + mgh + \frac{1}{2} m v^2 = \text{constant}$$

$$P \cdot A \cdot \Delta x = F \cdot \Delta x$$

11. Periodic Motion & Waves

1. Hooke's Law of spring

If a spring has resting length l_0 , and it's stretched or compressed to length $(l_0 + x)$, then it exerts a force:

$$F_{\text{spring}} = k \cdot x$$

* k : spring constant N/m

F_{spring} direction is opposite the direction of pull or push.

2. Potential energy of springs:

$$E_p = \frac{1}{2} k \cdot x^2$$

3. Conservation of Energy

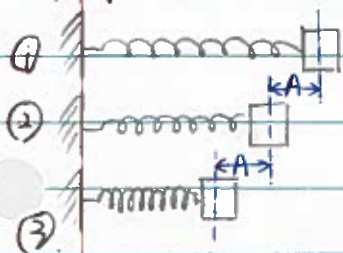
$$E_k + E_p = \text{constant} \quad \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \text{constant}$$

eg: One end of a horizontal spring ($k = 20 \text{ N/m}$) is connected to a wall, and the other end is connected to a mass $m = 0.2 \text{ kg}$. The spring is compressed 0.1 m from equilibrium.

After it's released, how much work does the spring do in order to push the mass to the equilibrium position?

$$W = \Delta E_k = -E_p = \frac{1}{2} k \cdot x^2 = 0.1 \text{ J}$$

4. Periodic Motion: One Oscillator



$$\textcircled{1} t=0 \quad x=A \quad v=0 \quad a=-\frac{kA}{m} \quad E_p = \frac{1}{2} kA^2 \quad E_k=0$$

$$\textcircled{2} t=\frac{1}{4}T \quad x=0 \quad v=-v_{\text{max}} \quad a=0 \quad E_p=0 \quad E_k = \frac{1}{2} m v_{\text{max}}^2$$

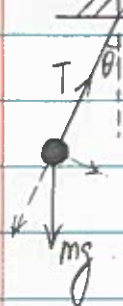
$$\textcircled{3} t=\frac{1}{2}T \quad x=-A \quad v=0 \quad a=\frac{kA}{m} \quad E_p = \frac{1}{2} kA^2 \quad E_k=0$$

$$\textcircled{2} t=\frac{3}{4}T \quad x=0 \quad v=v_{\text{max}} \quad a=0 \quad E_p=0 \quad E_k = \frac{1}{2} m v_{\text{max}}^2$$

$$\textcircled{1} t=T \quad x=A \quad v=0 \quad a=-\frac{kA}{m} \quad E_p = \frac{1}{2} kA^2 \quad E_k=0$$

5 Frequency of a spring: $f = 1/T = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Frequency of a pendulum: $f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$



$$F_{\perp} = mg \sin \theta \quad \text{当 } \theta \rightarrow 0 \text{ 时, } \sin \theta \rightarrow \theta \\ = mg \cdot \theta$$

6 Waves: $f = 1/T$ $v = \lambda \cdot f$

water wave

longitudinal & transverse

wave on plucked string

transverse

sound

longitudinal

earthquake

longitudinal & transverse

light

transverse

7 Interference & Node & Antinode

(1) Constructive interference

two waves are "in phase"



(2) Destructive interference

two waves are "out of phase"



* Constructive interference occurs when 2 waves differ by no wavelengths, or 1, 2, 3, ...

Destructive interference occurs when 2 waves differ by $1/2$ wavelengths, or $3/2$, $5/2$, ...

8 Standing Waves. * have only certain allowed f .

* For a string held at both ends.

① fundamental.  $l = \frac{1}{2}\lambda$

② second harmonic.  $l = \lambda$

③ third harmonic.  $l = \frac{3}{2}\lambda$

12. Sound

1 Intensity & Pitch

① Intensity $I = \frac{\Delta E}{A \cdot \Delta t} = \frac{P}{A} \text{ W/m}^2$

* $A = 4\pi r^2$ 球形表面积

$I_0 = 10^{-12} \text{ W/m}^2$ 为 human ear barely perceptible intensity

② $\beta = 10 \times \log_{10} \frac{I}{I_0}$

* $\Delta(I/I_0) = 10^n$ 则 $\Delta\beta = 10n$


③ power going through surface A = power going through surface B.
 $P_A = P_B$


$$I_A = \frac{P_A}{4\pi r_A^2} \quad I_B = \frac{P_B}{4\pi r_B^2}$$


$$4\pi r_A^2 \cdot I_A = 4\pi r_B^2 \cdot I_B \quad \therefore I_A = \left(\frac{r_B}{r_A}\right)^2 I_B$$

2 Resonating Cavities

For one end closed, one end open pipe:

① Fundamental:  $\frac{1}{4}\lambda = L$

② Second harmonic:  $\frac{3}{4}\lambda = L$

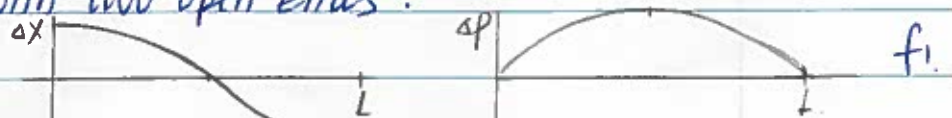
③ Third harmonic:  $\frac{5}{4}\lambda = L$

④ n. harmonic $\left[\frac{1}{4} + \frac{1}{2}(n-1)\right]\lambda = L$ ~~$\frac{n}{4}\lambda = L$~~

For open pipe with two open ends:

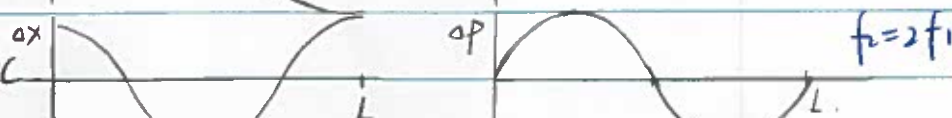
① Fundamental

$$\frac{1}{2}\lambda = L$$



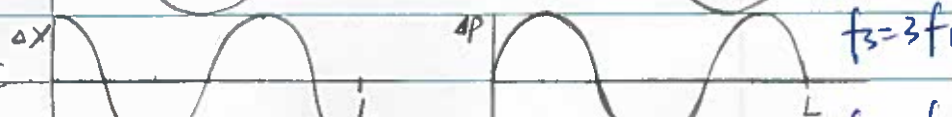
② Second harmonic

$$\lambda = L$$



③ Third harmonic

$$\frac{3}{2}\lambda = L$$



④ n harmonic $[\frac{1}{2} + \frac{1}{2}(n-1)]\lambda = L$

$$f_n = n f_1$$

3. Beats. $f_{\text{beat}} = |f_1 - f_2|$

- * ① Firstly 2 similar f waves are in phase.
- ② Then gradually out of phase.
- ③ Then in phase again.

4. Doppler Shift.

When the emitter of a wave & the detector are moving relative to each other, the detector detects a different f. from the one emitted. The f is higher if they are coming together, lower if they are going apart.

Application: $f_{\text{det}} = f_{\text{em}} \cdot \frac{V_s \pm V_{\text{det}}}{V_s \pm V_{\text{em}}}$

A police officer uses an emitter emits a $f_{\text{em}} = 60 \text{ kHz}$ to determine the speed of an approaching car ($V_{\text{car}} = 38 \text{ m/s}$)

The speed of sound is 343 m/s

- ① What f would the car detect if it could detect the emitter?
- ② What f would the officer detect from the reflection?

① $f_{\text{car}} = f_{\text{em}} \cdot \frac{V_s + V_{\text{car}}}{V_s}$

② $f_{\text{det}} = f_{\text{car}} \cdot \frac{V_s}{V_s - V_{\text{car}}}$

13. Light

1 Radio wave \rightarrow Microwave \rightarrow Infrared \rightarrow visible light \rightarrow Ultraviolet \rightarrow X \rightarrow γ rays
 $\lambda \downarrow$ $f \uparrow$

2 Light speed: ① in vacuum: $c = 3 \times 10^8 \text{ m/s}$

② non vacuum: $v = c/n$ * n : index of refraction.

Snell's Law: $n_i \sin \theta_i = n_f \sin \theta_f$

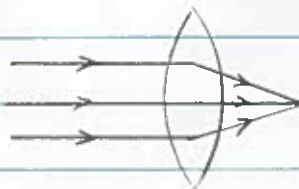
critical angle: $\theta_f = 90^\circ$ at θ_i

$$\sin \theta_c = \frac{n_f}{n_i}$$



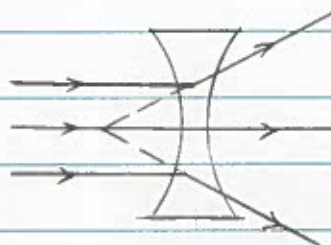
3

Converging Lens



$$f > 0$$

Diverging Lens:



$$f < 0$$

d_o : distance from lens \sim object. d_i : distance from lens \sim image

m : magnification of the image

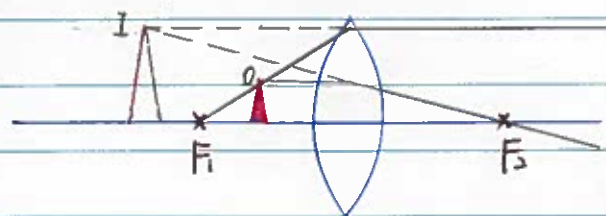
$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$m = -\frac{d_i}{d_o}$$

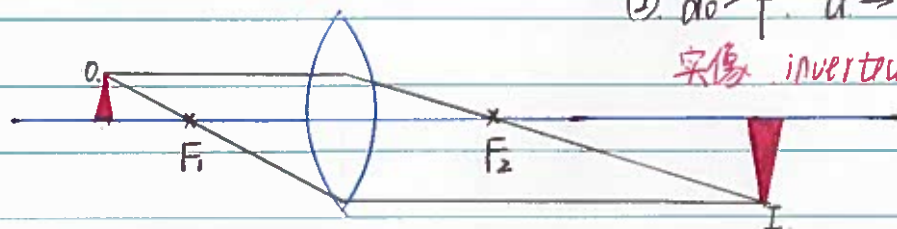
* $d_i < 0$ image & object @ same side $d_i > 0$ image @ the other side

* $m < 0$ image is inverted. $m > 0$ image is upright

4. Ray tracing for converging lens:

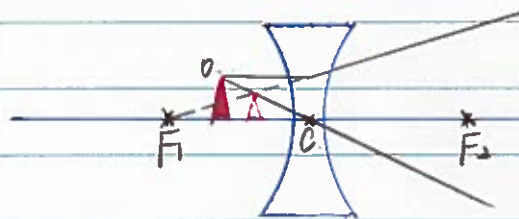


- ① $d_o < f$
- a. $F_1 O \rightarrow \text{lens} \rightarrow \text{平行线}$
 - b. $O \text{ 平行线} \rightarrow \text{lens} \rightarrow \text{过 } F_2$
 - c. 延长线交点即为 I .
- 虚像
upright

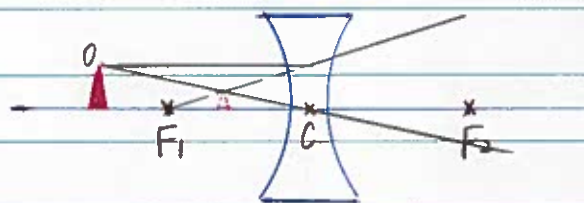


- ② $d_o > f$ a \rightarrow b \rightarrow c.
- 实像 inverted

5. Ray tracing for diverging lens:



- a. $O \text{ 平行线} \rightarrow \text{lens} \rightarrow \text{反延过 } F_1$
- b. $O C \text{ 直线过 lens}$
- c. 反延与 $O C$ 交点为 I .



- ① $d_o < f$ upright, virtual image
- ② $d_o > f$

6. Mirrors.

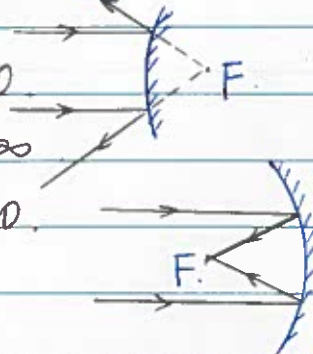
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = -\frac{d_i}{d_o}$$

* convex mirrors $f < 0$.

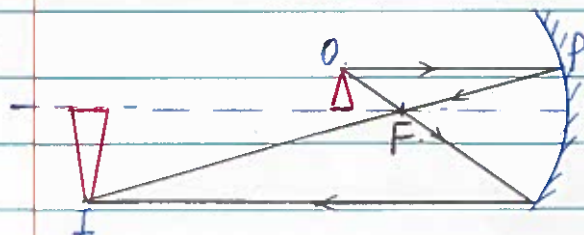
plane mirrors $f = \infty$

concave mirrors $f > 0$.



* $d_i > 0$ image & object @ same side.

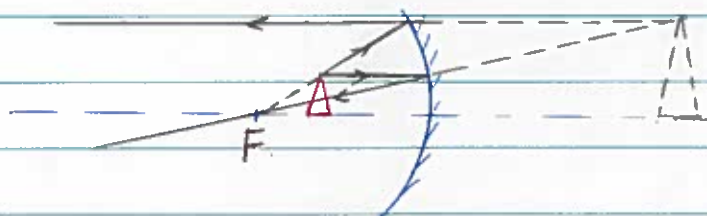
① Ray tracing for converging mirrors.



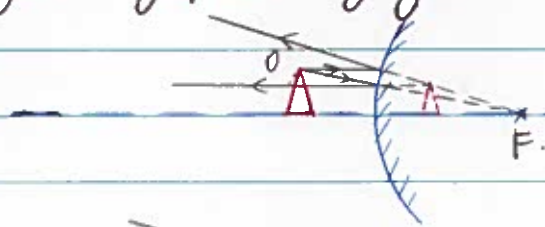
a. 从O画平行线 \rightarrow mirror \rightarrow 反射光线过F.

b. 画光线OF. 反射光线平行.

c. 延长线交点为I.



② Ray tracing for diverging mirrors.



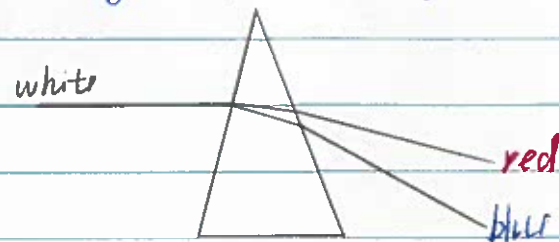
a. 从O画平行线 \rightarrow mirror \rightarrow 反射光线延长线过F.

b. 从O画光线延长过F. 反射光线为平行光线.

c. 延长线交点为I.



7 Dispersion: Different colors refract slightly differently, causing a separation of colors in a prism.

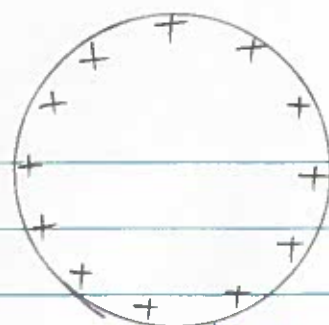


8 Combination of Lenses

$$\frac{1}{f_{\text{total}}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots$$

$$\text{power of lens } m^{-1} = D = \frac{1}{f}$$

14. Electrodynamics.



1. Charge & Materials.

① Conducting material: if given some charges, will evenly distributed on the surface; A charged object can induce a charge in a conductor.

② Nonconducting material: The induced charge in the nonconducting object is less than in a conductor.



2. Coulombs Law.

$$F_{\text{coul}} = \frac{k \cdot q_1 \cdot q_2}{d^2}$$

$$\times k = 9 \times 10^9 \text{ N.m}^2/\text{C}^2$$

$$F_{\text{grav}} = \frac{G M_1 M_2}{d^2}$$

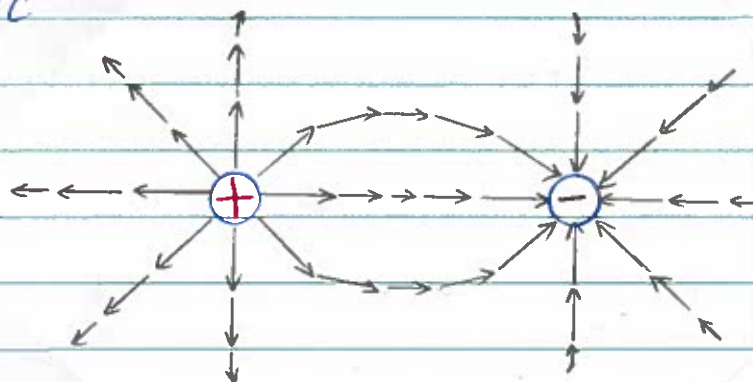
3. Electric Field.

$$E = \frac{k \cdot Q}{d^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

A charge q placed @ point P. will experience a force:

$$\vec{F} = q \cdot \vec{E}$$



4. Electric Potential: 电势.

For a lone charge Q . The potential at point P.

$$V = k \frac{Q}{d}$$

unit: J/C = volts = V.

If there are several charges: Q_1, Q_2, \dots

$$V = \frac{kQ_1}{d_1} + \frac{kQ_2}{d_2} + \dots$$

5. The work required to bring a charge from infinity to point P.

$$W = q \cdot V_p$$

The work required to move a charge q from A to B:

$$W = q \cdot \Delta V_{AB} = q \cdot (V_B - V_A)$$

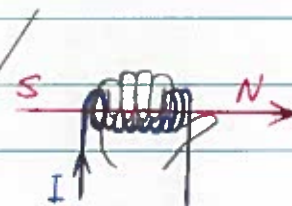
6. Magnetic Fields

① 右手定则:



I 方向为 \oplus 方向

判断感应电流方向



右手螺旋定则

伸开右手, 拇指与四指垂直且在手心平面, 让磁感线从手心进入, 拇指指向导线运动

四指即为感应电流

② 左手定则: 判断安培力的方向

伸开左手, 拇指与四指垂直且与掌在同一平面, 让磁感线从掌心进入

四指指向电流方向, 拇指方向即为通电导线在磁场中所受安培力的方向

③ 判断移动带电粒子在磁场中受力方向:

④ 右手半握拳 (四指与手掌垂直) 拇指指向粒子运动方向, 四指指向磁场方向, 手掌 \rightarrow 前臂方向为受力方向

⑤ 用左手

Electric Circuits

1 Ohm's Law: $\Delta V = I \cdot R$
 $V \quad A \quad \Omega$

2 For several resistors in series: $R_T = R_1 + R_2 + \dots$

For several resistors in parallel: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$

3 Real DC cells $\Delta V = V_{emf} - I \cdot R_{int}$

Real Wires $R = \rho \frac{L}{A}$

$\times L$: length of the wire. m

A : cross-sectional area m^2

ρ : $\Omega \cdot m$

4 Power: $P = I \cdot \Delta V$ $J/s = \text{Watts}$

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

when comparing 2 bulbs which one is brighter, we are comparing P .

5 Capacities: $Q = C \cdot \Delta V$

Coulombs Farads Volt

① $W = q \cdot \Delta V$ the work needed to move a charged q across a capacity

② $W = F_{elec} \cdot \Delta x$

③ $F_{elec} = q \cdot E$

$\therefore \Delta V = E \cdot \Delta x$

E : magnitude of the electric field inside of the capacity.

Atomic & Nuclear Physics

1. $q_{\text{proton}} = +1.6 \times 10^{-19} \text{ C}$

2. $E_n = -\frac{R_y}{n^2}$ $\frac{-2.18 \times 10^{-18}}{n^2}$

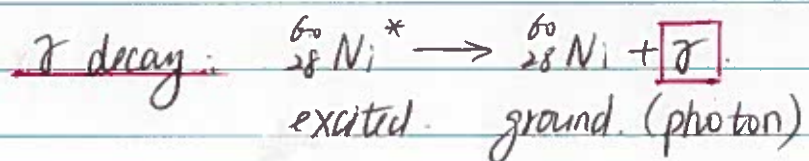
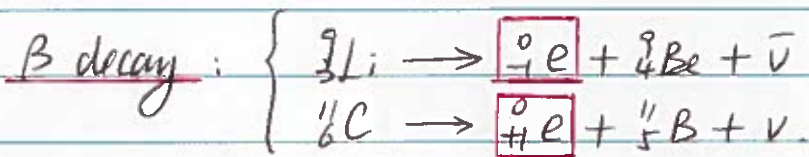
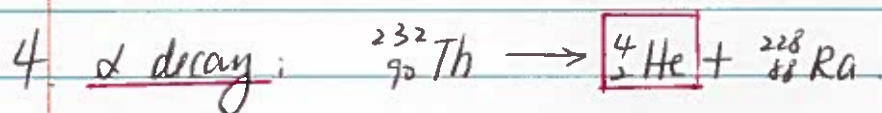
For an isolated atom, it only has certain energies according to the upper equation.

3. $E_{\text{photon}} = h \cdot f$

* $h = 6.63 \times 10^{-34} \text{ J/Hz}$ Planck's constant

$c = \lambda \cdot f$

* $c = 3 \times 10^8 \text{ m/s}$



5. $E = m_{\text{def}} \cdot c^2$

* $m_{\text{def}} = m_{\text{reactant}} - m_{\text{product}}$